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The author will welcome any criticism regarding any part of the manuscript. Only from such criticism can technology be improved.

NOMENCLATURE

α Dimensionless numerical factor (See Appendix I)

β Dimensionless numerical factor (See Appendix I)

γ Density of the content, lb/in^3 (kg/m^3)

σ Stress, psi (MPa)

σ_{max} Maximum stress, psi (MPa)

a Width of tank, in (m)

A Cross-sectional area, in^2 (m^2)

b Length of tank, in (m)

d Deflection, in (mm)

d_{max} Maximum deflection, in (mm)

D Depth, in (m)

E Modulus of elasticity, psi (MPa)

h Height of tank, in (m)

I Moment of inertia, in^4 (m^4)

I_{req} Required moment of inertia, in^4 (m^4)

l Length, in (m)

M Bending moment, lb-IN (N-m)

M_{max} Maximum bending moment, lb-IN (N-m)

P Pressure, psi (KPa)

R_b Bottom edge reaction, lb/IN (N/m)

R_h Intermediate horizontal stiffener reaction, lb/IN (N/m)

R_t Top edge reaction, lb/IN (N/m)

S Allowable stress, psi (MPa)

S_y Minimum yield strength, psi (MPa)

t	Plate thickness, in (mm)
t_a	Actual plate thickness to be used, in (mm)
t_r	Required plate thickness, in (mm)
W	Unit load, lb/IN (N/m)
x	Length of rectangular plate, in (m)
y	Width of rectangular plate, in (m)
Z	Section modulus, in^3 (m^3)
Z_{req}	Required section modulus, in^3 (m^3)

A SIMPLE METHOD FOR THE DESIGN OF RECTANGULAR STORAGE TANKS

1.0 INTRODUCTION

The purpose of this analysis is to describe a simple method for the design of a rectangular non-pressurized storage tank which will meet all the design requirements stated in the following paragraphs.

Tanks are usually designed and fabricated according to the minimum requirements of certain specifications or codes. The current API Codes do not have any specific requirements for rectangular storage tank design or fabrication. Recently, the ASME Boiler and Pressure Vessel Code, Sectin VIII Division 1, 1979 Summer Addenda issued a new set of mandatory rules (Appendix XIII) for the design of rectangular and obround vessels. They may be used for reference.

Rectangular storage tanks are usually designed in accordance with the following criteria:-

i) Rectangular in shape

ii) For nonpressurized storage

iii) Tank wall is subject to hydrostatic head only (i.e. uniformly increasing load from top to bottom and normal to its plane)

iv) No external pressure

v) Design temperature:- from room temperature up to 200°F

vi) Corrosion allowance to be included in the design, if required.

In the following chapters, the basic design formulae for thin plates under bending and membrane tension, also the effect of structural stiffeners and the stress analysis in some cases are discussed. The most common tank wall configurations, unstiffened and stiffened are analysed and their applications are illustrated in a numerical example .

2.0 SCOPE

The design of rectangular storage tanks is based on the stress analysis of flat rectangular plates under different cases of loading and edge conditions.

The bending properties of a flat plate depend largely on its thickness as compared to its other dimensions. Plate theory usually distinguishes three different categories:

i) Thin plates with small deflection
ii) Thin plates with large deflections
iii) Thick plates.

2.1 Thin Plates With Small Deflections

If the deflection of a plate is small in comparison with its thickness, a very satisfactory approximate theory of bending of the plate by lateral loads can be developed by making the following assumptions:

i) There is no deformation in the middle plane of the plate. This is often called the neutral plane.
ii) Points of the plate lying initially in a normal-to-the-middle plane of the plate remain on the normal-to-the-middle plane of the plate after bending. The effect of shear forces on the deflection of the thin plates is considered negligible.
iii) The normal stress in the direction transverse to the plate can be disregarded.

All stress components can be expressed by the deflection of the plate, which is a function of the two coordinates in the plane of the plate. This function can be expressed as a linear partial differential equation. For a given set of boundary conditions, the stresses at any point of the plate can be calculated. For rectangular plate analysis, these equations are very well explained in many plate theory books [8].

2.2 Thin Plate With Large Deflection

When the deflection becomes larger than about one-half the thickness, there is strain in the middle plane. The stress in this middle plane cannot be ignored. This stress, called diaphragm stress or direct stress, enables the plate to carry part of the load as a diaphragm in direct tension. This tension may be balanced by radial tension at the edges if the edges are held or by circumferential compression if the edges are not horizontally restrained.

Membrane stresses would also arise in such a plate if its edges are immovable in its plane and the deflections are sufficiently large.

If all these are taken into consideration in deriving the differential equation, a nonlinear load-deflection and load-stress relation is obtained. However, for rectangular plates under uniform load producing large deflections, analytical solutions which relate load, deflection and stress in terms of numerical values of the dimensionless coefficients: d/t, Pb^4/Et^4 and $\sigma b^2/Et^2$ can be obtained from [5].

2.3 Thick Plates

The approximate theories of thin plates becomes unreliable in the case of plates of considerable thickness, especially in the case of highly concentrated loads. In such a case, the thick-plate theory should be applied. This theory considers the problem of plates as a three-dimensional problem of elasticity.

2.4 Thin Plate Theory For Rectangular Tank Design

The formulae used in this technical report are based on approximate mathematical analysis of small deflection-thin-plate theory. All these solutions of numerical analysis are obtained from [5], [6], [7] and [8] and can be accepted as sufficiently accurate so long as the following assumptions hold true:

i) The plate is flat, of uniform thickness and of homogeneous isotropic material.

ii) The plate thickness is not more than about one-quarter of the least transverse dimension.

iii) The maximum deflection is not more than about one-half of the plate thickness.

iv) The loadings are normal to the plane of the plate.

v) The plate is nowhere stressed beyond the elastic limit (i.e. Theory of linear elasticity is applied).

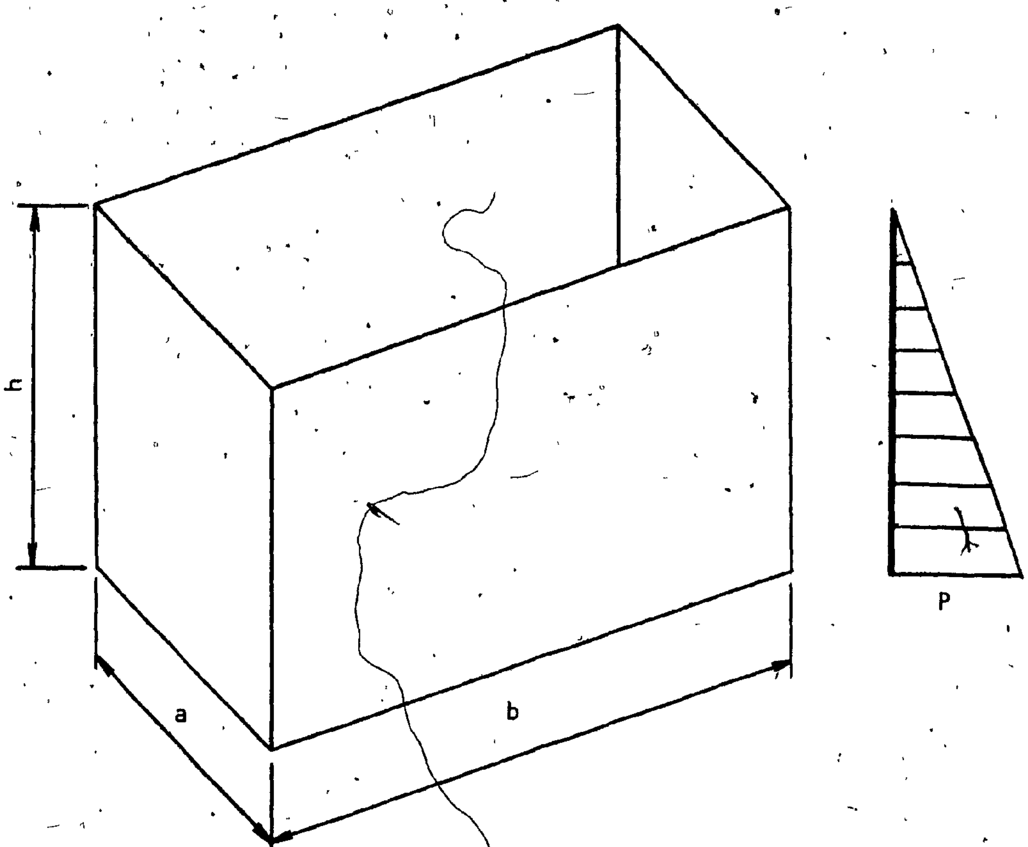


Figure 1. Rectangular Storage Tank

3.0 DESIGN OF TANK SIDE WALLS WITHOUT STIFFENERS

The tank wall is subjected to an uniformly increasing load which varies from top to bottom edge. This internal pressure is due to the weight of the contents. The hydrostatic pressure can be expressed as:- (See Figure 1)

P = maximum hydrostatic pressure

$$= h\gamma \tag{1}$$

where

h = the maximum height of the contents, height of the tank or height to overflow nozzle

γ = density of the contents

For conservative design, it is simply to analyse only the longest side wall, having the greatest span between supports. Tanks without stiffeners will have their top edge free and the remaining three edges simply supported. A typical tank wall is illustrated in Figure 2. The maximum bending stress and deflection on the wall can be determined by applying flat plate formulas with the above loading and edge conditions. These maximum stress and deflection are given by

σ_{max} = maximum bending stress in the plate

$$= \beta P b^2/t^2 \tag{2}$$

d_{max} = maximum deflection of the plate

$$= \alpha P b^4/Et^3 \tag{3}$$

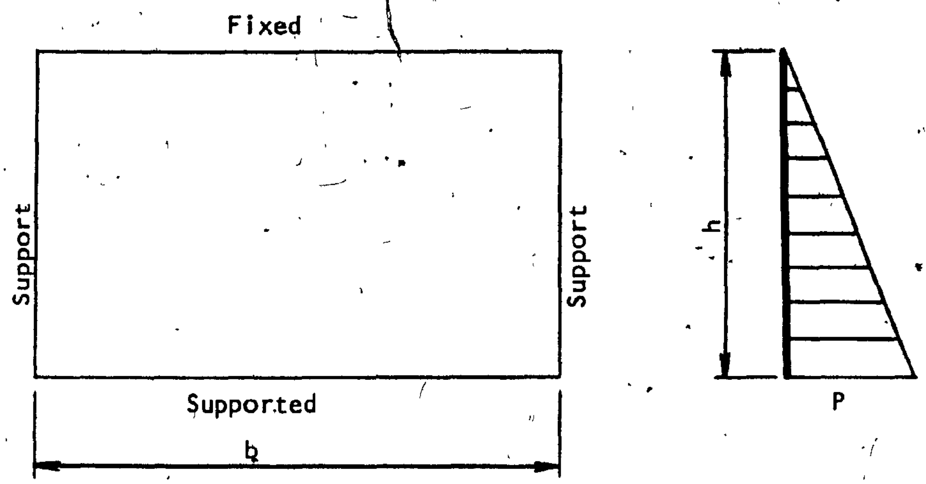


Figure 2. Wall Panel Without Stiffeners

where

α = dimensionless numerical factor depending on the ratio height, h; to the length, b (i.e. h/b). Values can be obtained from Graph #1

β = dimensionless numerical factor, see Graph #1

t = plate thickness

E = modulus of elasticity

By rearranging equation (2), the required wall thickness is:-

t_a = minimum required wall thickness

$$= b\sqrt{\frac{\beta P}{S}} \qquad (4)$$

where

S = maximum allowable stress

Corrosion allowance should be considered in deriving the actual plate thickness t_a. However, in no case will the minimum required wall thickness be less than 3/16 inch for all carbon steel tanks as recommended by API codes. Generally, stainless steel tanks may be designed with less than 3/16 inch thick wall panel.

The actual plate thickness t_a (less corrosion allowance) can be employed in equation (3) to determine the maximum deflection. The maximum deflection should not exceed one half of the required plate thickness.

The available numerical data, as shown on Graph #1 are limited for values of h/b from 0.5 to 5. For extreme cases, such as, very long ($h/b \to 0$) and very high ($h/b \to \infty$) rectangular tanks which are very seldom used, flat plate theory may be replaced by straight beam analysis.

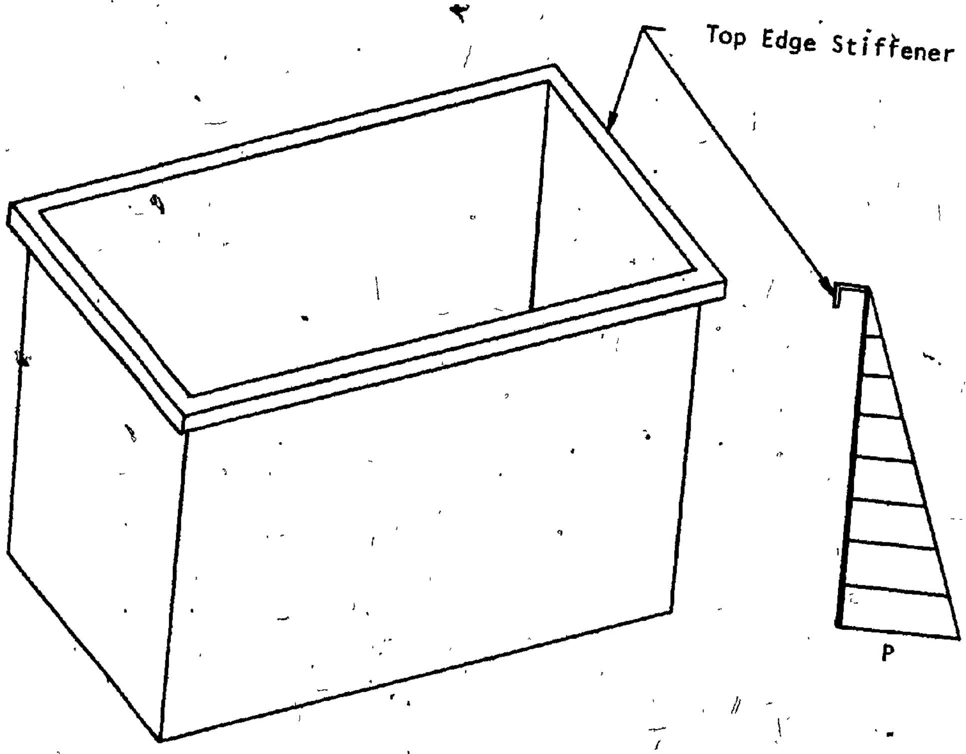


Figure 3. Tank With Top-Edge Stiffener

4.0 DESIGN OF TANK SIDE WALLS WITH TOP EDGE STIFFENERS

If the plate thickness as determined by the previous section seems uneconomical, or the plate deflection is too large and exceeds one half of its thickness, a top edge stiffener should be added.

Top edge stiffeners are required for open-top tank. Stiffeners shall be located at the top and preferably on the outside of the tank shell. In this case, all edges of the tank wall may be considered to be simply supported. The minimum wall thickness and the maximum deflection can be obtained by equations (3) and (4) with appropriate values of α and β from Graph #2 for $h < b$ or from Graph #3 for $h > b$. (See Figure 3.)

For sizing the top edge stiffeners of small tanks, straight beam analysis may be used without taking into account the corner moments at the top edge frame.

For a top and bottom supported tank wall, the top and bottom edge reactions can be expressed as

R_t = top edge reaction per unit length

$$= Ph/6 \tag{5}$$

R_b = bottom edge reaction per unit length

$$= Ph/3 \tag{6}$$

The top edge stiffener is subjected to the uniformly distributed load R_t. The maximum deflection of the beam for this load is given by

$$d_{max} = \frac{R_t b^4}{384EI} \tag{7}$$

Since this maximum deflection of the panel is limited to one half of the plate thickness, we have

$$\frac{t_a}{2} = \frac{R_t\ b^4}{384EI} \tag{8}$$

By rearranging equation (8), the size of the top edge stiffeners can be determined from its minimum required moment of inertia. That is

$$I_{req} = \frac{R_t\ b^4}{192Et_a} \tag{9}$$

For an economical design, a rigid frame with corner moment connections should be used rather than simply supported beams. Since the top edge stiffeners form a closed rectangular frame with moment connections at the corners, the stiffeners can be analysed as rigid frame. (See Figure 4.)

By applying the three-moment-equation, the corner moment and the maximum moment at each span (using a constant moment of inertia for all sides of the frame) can be obtained as follows

M_c = corner moment

$$= \frac{R_t}{12} \times \frac{b^3 + a^3}{b + a} \tag{10}$$

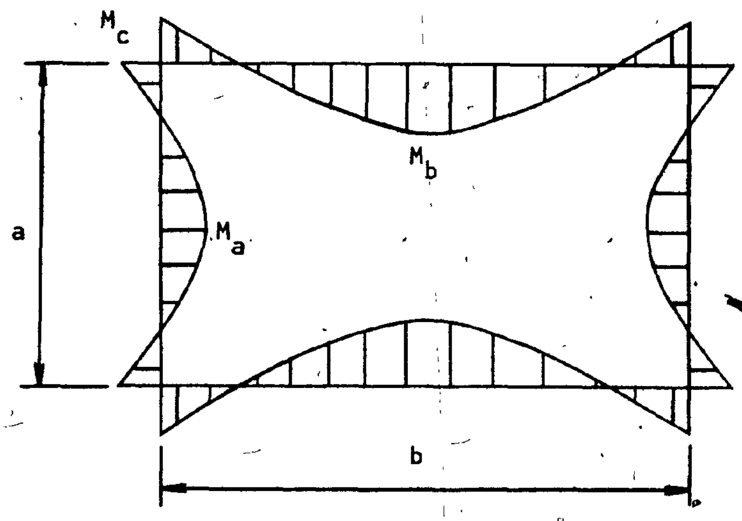


Figure 4. Bending Moment diagram of Top edge - Stiffeners (Rigid Frame)

M_b = maximum bending moment on length side of stiffener

$$= \frac{R_t\ b^2}{8} - M_c \qquad (11)$$

M_a = maximum bending moment on width side of stiffener

$$= \frac{R_t\ a^2}{8} - M_c \qquad (12)$$

Then, the bending stresses at the above locations of the top edge stiffeners can be expressed as

σ_c = bending stress at corner

$$= \frac{M_c}{Z} \qquad (13)$$

σ_b = maximum bending stress at length side of stiffener

$$= \frac{M_b}{Z} \qquad (14)$$

σ_a = maximum bending stress at width side of stiffener

$$= \frac{M_a}{Z} \qquad (15)$$

where

Z = section modulus of stiffener

These bending stresses must be less than the allowable stress for the given material. For structural members, CSA Standard S16 - Steel Structures for Buildings is recommended as a guideline.

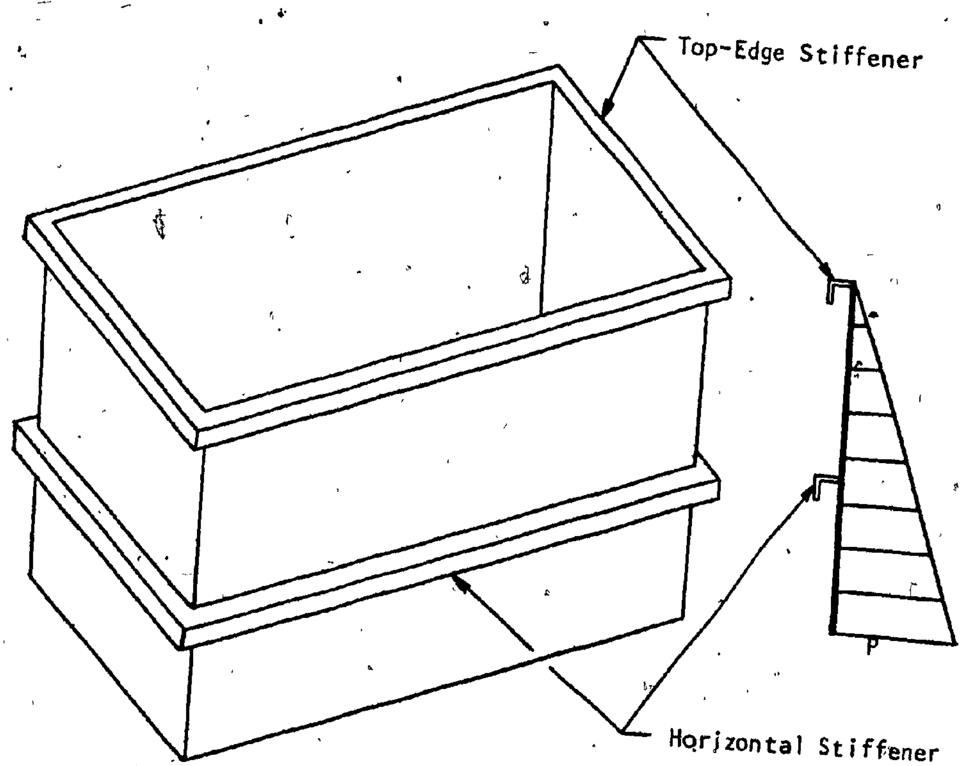


Figure 5. Tank With Horizontal Stiffener

5.0 DESIGN OF TANK SIDE WALLS WITH HORIZONTAL STIFFENERS

The plate thickness can be reduced considerably by adding horizontal or vertical stiffeners or a combination of both. The addition of stiffeners will increase the rigidity of the tank wall by increasing the moment of inertia of the combined section.

For the design of tank walls with intermediate horizontal stiffeners, there is no simple formula available. For a conservative design, wall panels may be analysed as straight beams by considering strips of unit width of the plate under the hydrostatic load.

The first step is to locate the stiffener. The optimum position is located at the point where the moment due to the distributed load above the stiffener is equal to that produced by the load below it. (See Figures 5 and 6.)

We have,

M_{ht} = fixed end moment due to distributed load above the stiffener

$$= -\frac{(mP)(mh)^2}{15} \qquad (16)$$

M_{hb} = fixed and moment due to distributed load below the stiffener

$$= -\frac{7(nP)(nh)^2}{120} - \frac{(mP)(nh)^2}{8} \qquad (17)$$

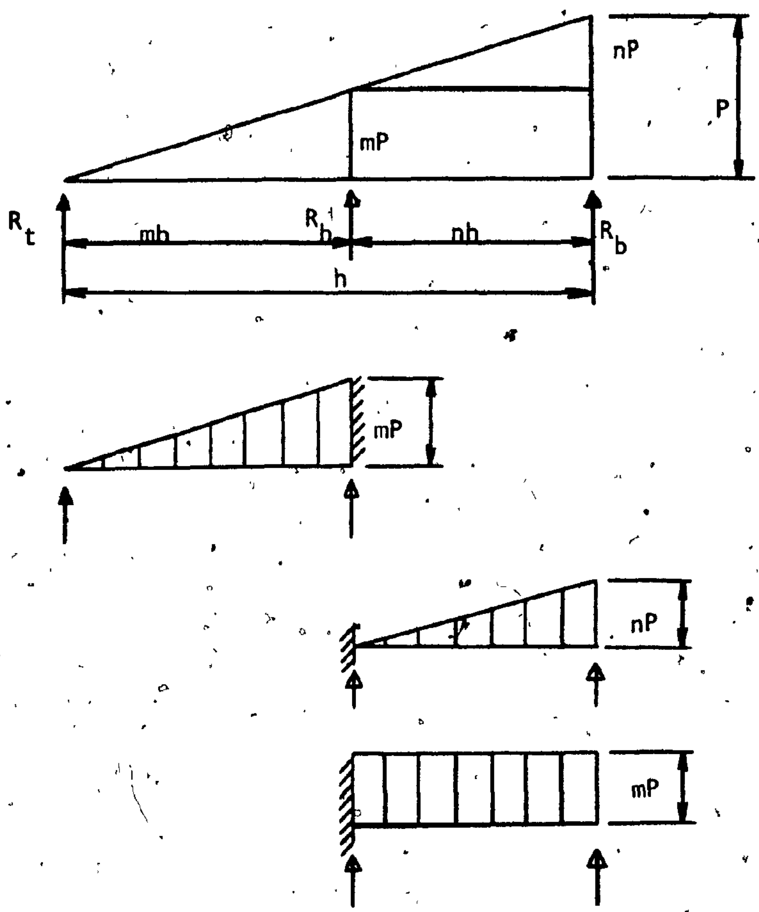


Figure 6. Loading Diagram of Intermediate
Horizontal Stiffener-Panel

By equating equations (16) and (17), we obtain:

$$\frac{Pm^3h^2}{15} = \frac{7Pn^3h^2}{120} + \frac{Pmn^2h^2}{8}$$

$$8m^3 = 7n^3 + 15mn^2 \tag{18}$$

also, we have

$$m + n = 1 \tag{19}$$

Solving equations (18) and (19) for m and n, we find

$$m = \frac{1}{3}(2\sqrt{2} - 1)$$
$$= 0.61$$

$$n = \frac{1}{3}(4 - 2\sqrt{2})$$
$$= 0.39$$

By substituting the values of m and n into either equation (16) or (17), we can obtain the bending moment at the intermediate horizontal stiffener in terms of P and h.

M_h = bending moment at intermediate horizontal stiffener

$$= M_{ht} = M_{hb}$$

$$= -\frac{Pm^3h^2}{15}$$

$$= -\frac{22\sqrt{2} - 25}{405} Ph^2$$

$$= -0.01509\, Ph^2 \tag{20}$$

To determine the top edge reaction R_t, the intermediate stiffener reaction R_h and the bottom edge reaction R_b, we have,

$$R_t mh = \frac{1}{6} Ph^2 m^3 - M_h$$

$$= \frac{1}{6} Ph^2 m^3 - \frac{P\ m^3\ h^2}{15}$$

$$= \frac{P\ h^2\ m^3}{10}$$

$$R_t = \frac{P\ h\ m^2}{10}$$

$$= 0.03715\ Ph \qquad (21)$$

For the intermediate horizontal stiffener reaction:

$$R_t h + R_h nh = \frac{1}{6} Ph^2$$

$$R_h nh = \frac{1}{6} Ph^2 - \frac{Ph^2 m^2}{10}$$

$$= \frac{Ph^2}{30} (5-3m^2)$$

$$R_h = \frac{Ph}{30n} (5-3m^2)$$

$$= 0.3316\ Ph \qquad (22)$$

For the bottom edge reaction:

$$R_b nh = \frac{1}{2} Ph^2 mn^2 + \frac{1}{3} Ph^2 n^3 - M_h$$

$$= \frac{1}{2} Ph^2 mn^2 + \frac{1}{3} Ph^2 n^3 - \frac{1}{15} Ph^2 m^3$$

$$R_b = \frac{Ph}{30n} (3m^3-15M+10)$$

$$= 0.1312\ Ph \qquad (23)$$

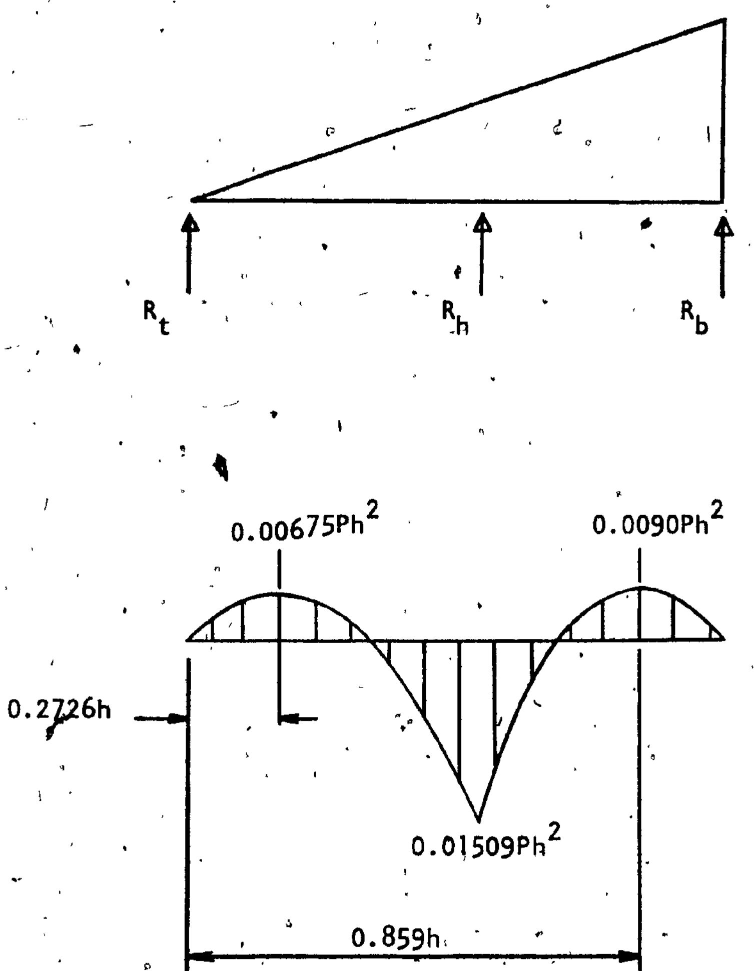


Figure 7. Bending Moment Diagram of Intermediate - Horizontal Stiffener-Panel

Then, we have to determine the maximum bending moment of each section above and below the immediate horizontal stiffener. (See Figure 7.) For the top section, $0 < kh < mh$

$$M = R_t kh - \frac{1}{6} Ph^2k^3$$

$$= \frac{1}{10} Ph^2m^2k - \frac{1}{6} Ph^2k^3$$

$$\frac{dM}{dk} = \frac{1}{10} Ph^2m^2 - \frac{1}{2} Ph^2k^2$$

For maximum bending moment, we have

$$\frac{dM}{dk} = 0$$

or

$$0 = \frac{1}{10} Ph^2m^2 - \frac{1}{2} Ph^2k^2$$

$$k = \frac{a}{\sqrt{5}}$$

$$= 0.2726$$

Hence, the maximum bending moment at the top section is:

$$M_{max} = \frac{Ph^2m^3}{10\sqrt{5}} - \frac{Ph^2m^3}{30\sqrt{5}}$$

$$= \frac{Ph^2m^3}{15\sqrt{5}}$$

$$= 0.00675\ Ph^2 \qquad (24)$$

For the bottom section, $mk < kh \leq h$

$$M = R_t\ kh + R_h\ h\ (k-m) - \frac{1}{6} Ph^2k^3$$
$$= \frac{1}{10} Ph^2m^2k + \frac{Ph^2k}{30n} (5-3m^2) - \frac{Ph^2m}{30n} (5-3m^2)$$
$$- \frac{1}{6} Ph^2k^3$$

$$\frac{dM}{dk} = \frac{1}{10} Ph^2m^2 + \frac{Ph^2}{30n} (5-3m^2) - \frac{1}{2} Ph^2k^2$$

For maximum bending moment, we have

$$\frac{dM}{dk} = 0$$

or

$$0 = \frac{1}{10} Ph^2m^2 + \frac{Ph^2}{30n} (4-3m^2) - \frac{1}{2} Ph^2k^2$$

$$k = 0.859$$

Hence, the maximum bending moment at the lower section is:

$$M_{max} = 0.0090\ Ph^2 \tag{25}$$

Finally, we can determine the required wall thickness. By equation (20), we have

$$\sigma_h = \text{bending stress at intermediate horizontal stiffener}$$
$$= 6\ M_h/t_r^{\ 2}$$
$$= 0.09054\ Ph^2/t_r^{\ 2} \tag{26}$$

Hence, by rearranging equation (26), the required tank wall thickness is

$$t_r = \text{required tank wall thickness}$$
$$= 0.301\, h \sqrt{P/S} \qquad (27)$$

The sizes of the top edge and the intermediate horizontal stiffeners can be approximately determined by applying equation (9) with corresponding values of R_t and R_h respectively.

For tanks with more than one intermediate horizontal stiffener, a similar approach of analysis as described above can be used. However, in determining the required wall thickness by equations (26) and (27), only the maximum value of the bending moments of all the intermediate horizontal stiffeners should be used.

6.0 DESIGN OF TANK SIDE WALLS WITH VERTICAL STIFFENERS

If the plate thickness required for unstiffened tanks is unrealistically high, then vertical stiffeners and sometimes a combination of horizontal and vertical stiffeners may be used.

In this case, the design procedure is broken down into several steps:

a) To determine the maximum stiffener spacing for a desired plate thickness.

b) To establish stiffener layout and spacing.

c) To determine size of vertical stiffener from panel loads.

d) To determine size of top and bottom support beams.

For the first step, the panel is considered in the form of multiple ribs. By applying straight beam analysis with both ends fixed and under an uniformly distributed load, the maximum moment can be expressed as

M_{max} = maximum bending moment

$$= Pl/12$$

and

σ = bending stress

$$= \frac{6\ M_{max}}{t^2}$$

$$= \frac{Pl^2}{2t^2}$$

Hence, the maximum stiffener spacing for a desired plate thickness is

l = maximum stiffener spacing for plate thickness t

$$= t\sqrt{\frac{2S}{P}} \tag{28}$$

If it is desired to limit the plate deflection to one half of its thickness, a similar expression can be obtained

d_{max} = maximum deflection

$$= \frac{P\ l^4}{384EI}$$

where

$$I = t^3/12$$

Hence, we have

$$d_{max} = \frac{P\ l^4}{32Et^3}$$

For $d_{max} = t/2$ we have

$$l = 2t\sqrt[4]{E/P} \tag{29}$$

For tanks of low to intermediate height, usually equation (28) in conjunction with maximum allowable stress applies. For high, reinforced tanks with vertical stiffeners only, the deflection expression, equation (29), may limit the stiffener spacing.

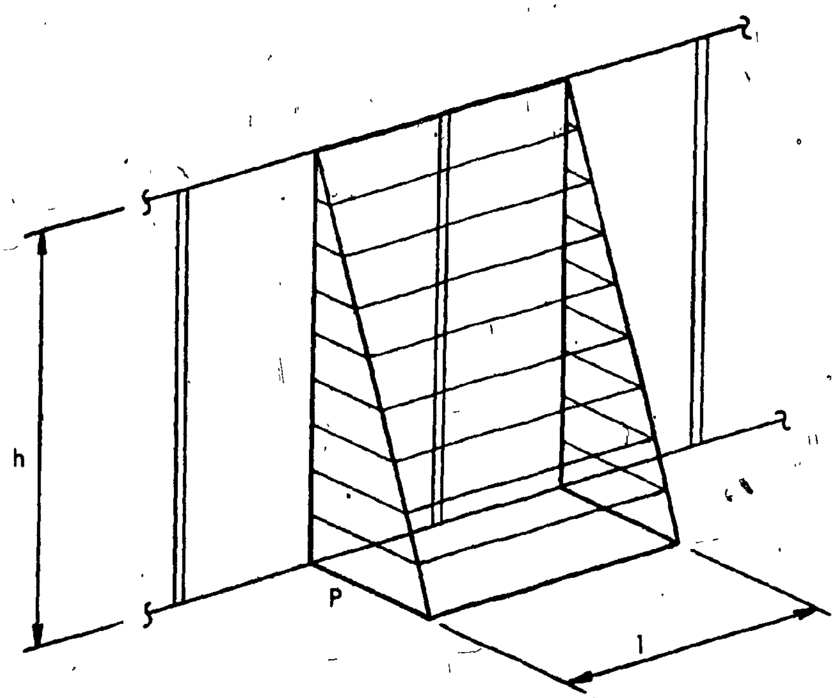


Figure 8. Wall Panel With Vertical Stiffeners

The addition of vertical stiffeners in accordance with the stiffener spacing as determined above, divides the wall panel into smaller sections. Each section of the wall panel can be considered as a plate under a uniformly increasing load with the two vertical edges fixed and the horizontal edges simply supported. For this loading condition, no convenient numerical data are available. Therefore, the case of top edge simply supported and other edges fixed is usually substituted. The maximum bending stress which should not exceed the allowable stress can be expressed as:-

$$\sigma_{max} = \text{maximum bending stress}$$

$$= \frac{\beta P\ b^2}{t^2}$$

Once the plate thickness and the stiffener spacing are determined, the stiffeners may be analysed as simple beams loaded by a uniformly increasing load. The maximum moment is given by (See Figure 8)

$$M_{max} = \text{maximum bending moment}$$

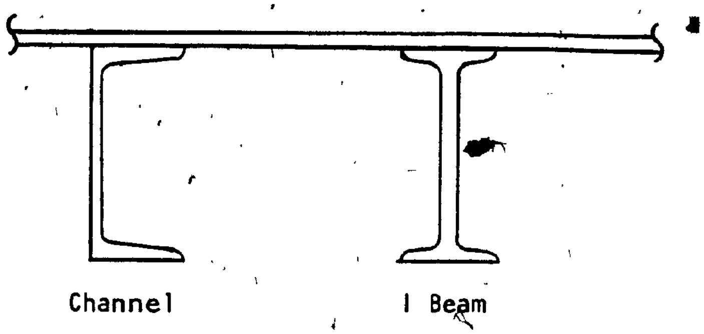
$$= 0.0641\ Plh^2 \qquad (30)$$

Hence, the required section modulus of the vertical stiffener is

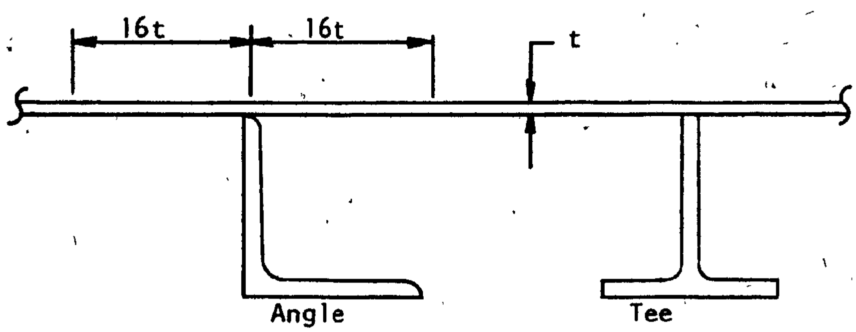
$$Z_{req} = \text{required section modulus}$$

$$= \frac{M_{max}}{S}$$

$$= 0.0641Plh^2/S \qquad (31)$$



Contribution of plate is small and can be neglected



Contribution of plate may be included.

Figure 9. Different Types of Stiffener-attachments

The selected vertical stiffeners should have a section modulus equal to or greater than Z_{req}. For rolled sections such as channels or I beams, the contributions of the plate are often neglected. However, for angles or flat rib sections, a width of 16 plate thickness on each side of the stiffener may be included in the available section modulus. (See Figure 9.)

Finally, for sizing of the top and bottom supporting beams, it is safe to assume that all loads transmitted through the vertical stiffener and the tank plate are evenly distributed along these supporting beams. The analysis is as described previously in Section 4.0.

7.0 DESIGN OF TANK BOTTOM PLATES AND ROOF PLATES

7.1 Tank Bottom Plate Design

Generally, a rectangular tank is either sitting on a flat base or is supported by beams. Whatever the support, the entire base plate is acted upon by a uniformly distributed hydrostatic load.

For the former case, the whole surface of the base plate is supported. No analysis is necessary. According to API Codes, a minimum thickness of 1/4 inch for carbon steel is recommended.

Regardless of the shape of the structural base, flat plate analysis with the appropriate loading and edge conditions is applicable. By applying equations (3) and (4), the minimum required plate thickness and the maximum deflection can be calculated. Dimension parameters, α and β can be obtained from appropriate Graphs #5 to 9 in Appendix I, depending on the edge conditions.

7.2 Roof Plate Design

As recommended by API 650, all roofs shall be designed to support a dead load, plus a uniform live load of not less than 25 lb. per sq. ft. of projected area. Roof plates shall have a minimum nominal thickness of 3/16 inch.

A similar approach as the bottom base plate analysis is applicable for the roof plate design. For self-supporting roofs, the minimum required thickness and the maximum deflection can be determined by applying flat-plate theory. The roof plate can also be stiffened by structural shapes welded to it.

In any case, corrosion allowance, if required, must be added to the required plate thickness to give the actual thickness to be used.

8.0 SUMMARY OF THE OVERALL DESIGN CONCEPTS

The efficient use of materials is the first essential of good design. One way to achieve such efficiency is to use lighter-gage plate, which is easily fabricated. Hence stiffeners are commonly employed to increase the rigidity of light-gage tank walls.

Even though vertical stiffeners are preferred, intermediate hrozontal stiffeners are recommended for tanks with a height of more than seven feet.

The economy of a design often depends on the availability of certain plate sizes and thicknesses if short deliveries are called for. In such a case even increased labour costs due to additional welding may be offset by the benefits gained from "on time" delivery.

In general, the use of lighter-gage plate is usually offset by more welding, hence higher labour cost. For most tanks, there exists some optimum plate thickness for which material plus fabrication costs are lowest. Any decrease of plate thickness beyond this optimum would increase the total cost of the tank due to increase labour cost.

The design method given in this paper is simple to apply and has been used successfully in practices.

9.0 A DESIGN REPORT TO ILLUSTRATE THE DESIGN METHOD

9.1 Design Conditions and Requirements

Medium	NAOH Solution
Density of NAOH Solution	93.35 lb/ft^3
Operating capacity	460 ft^3 (13 m^3)
Total volume	460 ft^3 (13 m^3)
Internal pressure	Atmospheric
External pressure	Nil
Design temperature	150°F (65°C)
Operating temperature	90°F max. (32°C)
Weight empty	7,700 lbs. (3500 kg)
Corrosion allowance	Nil

Material Specifications:-

Plates:- Austentic Stainless Steel
ASTM A-240-Type 316L
S = allowable stress
= 18840 psi (See Section 9.2)

Stiffeners:- Structural Steel
ASTM A-36 or equivalent
S = allowable stress
= per CSA S16 Standard for main support beams
or
= 18000 psi for secondary stiffeners etc.

Base plate is supported on (6) W6x15.5 beams at locations as shown in Figure 10.

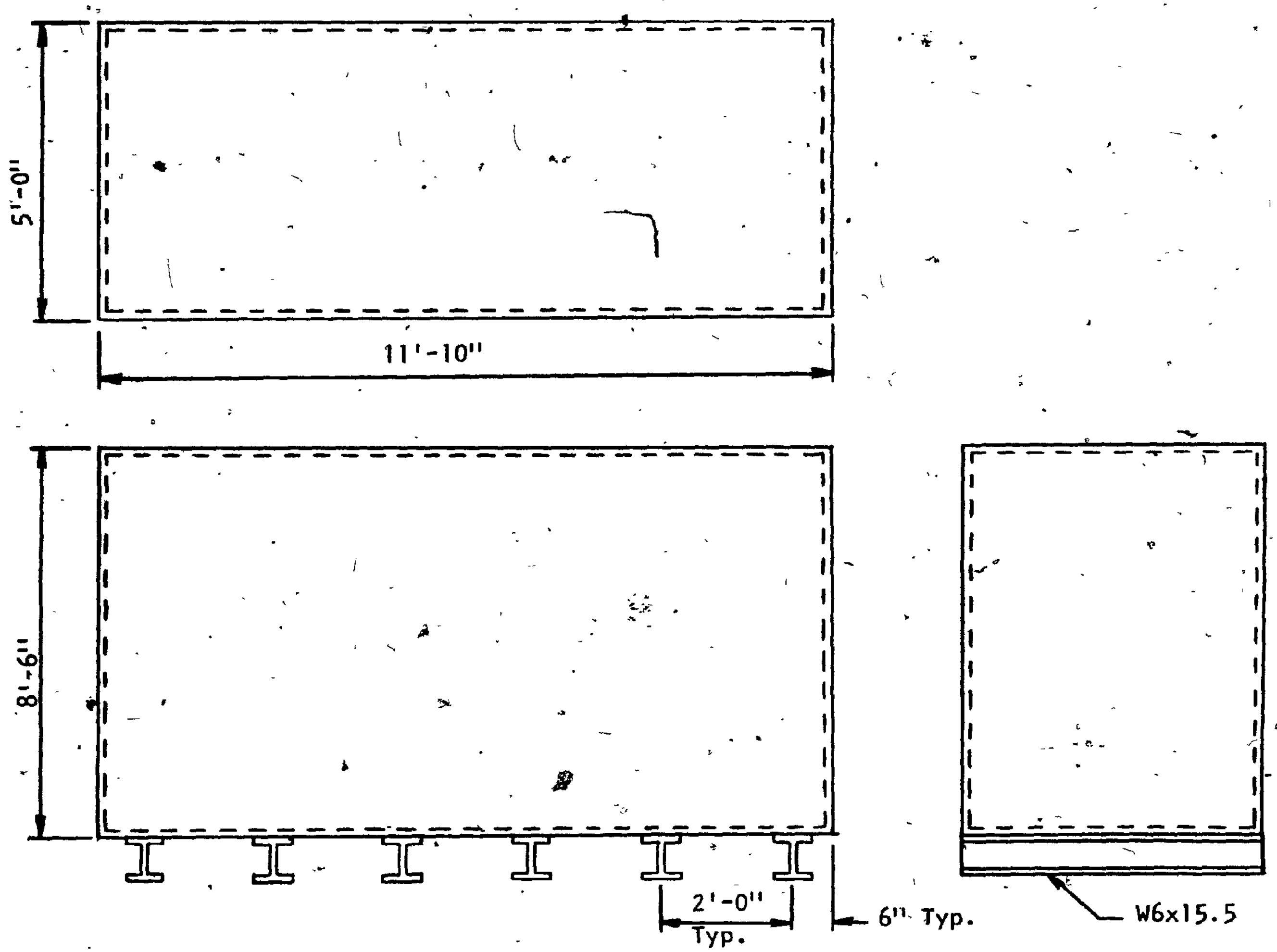


Figure 10. NaOH Solution Storage Tank

Tank Dimensions:-

h = 102 in.

a = 60 in.

b = 132 in.

9.2 Allowable Stresses

Stresses as recommended by the ASME Code for pressure vessels may be increased when used in designing tanks under hydrostatic pressure only. The maximum allowable stress is the lower of one-quarter of the ultimate tensile strength or five-eights of the minimum specified yield strength of the material.

However, for circular storage tank per API 650, an allowable stress of 21,000 psi for all materials is suggested for membrane stresses only. For all other tanks, also elevated circular storage tanks where high bending stresses may exist, the criteria of API 620 should be used. Allowable stress per API 620 are limited to 30/25 of allowable stresses per ASME Code Section VIII Division 1.

For structural members, allowable stresses per CSA S16 should be used.

9.3 Design Calculation

9.3.1 Hydrostatic Pressure at Tank Bottom

$$P = h\gamma$$
$$= 102 \times 93.35 \times 1/1728$$
$$= 5.5 \text{ psi}$$

9.3.2 Selection of Plate Thickness - Stiffener Spacing

$$l = t\sqrt{\frac{2S}{P}}$$

$$= t\sqrt{\frac{2x\ 18840}{5.5}}$$

t	1/8	3/16	1/4	5/16	3/8
l	10.35	15.52	20.69	25.87	31.04

Since the height of tank just exceed 7 ft., one horizontal intermediate stiffener at location 5 ft. from top is used.

9.3.3 Design of Tank Wall

a) Lower Panel

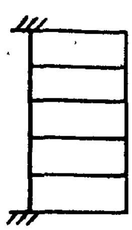
Try using 3/16" TK. plate with 8 panels spaced at 16 1/2".

The hydrostatic pressure at horizontal intermediate stiffener is:

$$P_h = 5.5 \times \frac{5.0}{8.5}$$

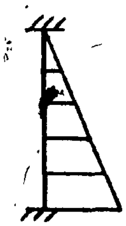
$$= 3.2 \text{ psi}$$

By principle of superposition, each panel can be considered subject to a uniform load and a triangular load.



$P_1 = 3.2$ psi

+



$P_2 = 2.3$ psi

$$\frac{h}{b} = \frac{42}{16.5}$$

$$= 2.54$$

$\beta_1 = 0.500$ (from Graph #9)

$\beta_2 = 0.270$ (from Graph #4)

σ_{max} = maximum bending stress

$$= \frac{\beta_1 P_1 b^2}{t^2} + \frac{\beta_2 P_2 b^2}{t^2}$$

$$= \left(\frac{b}{t}\right)^2 (\beta_1 P_1 + \beta_2 P_2)$$

$$= \left(\frac{16.5}{0.1875}\right)^2 (0.500 \times 3.2 + 0.270 \times 2.3)$$

$$= 17200 \text{ psi}$$

$\sigma_{max} < S$

i.e. $17200 < 18840$

Condition Satisfied.

Check with average load

$$P_{av} = 1/2\ (P + P_1)$$

$$= 1/2\ (5.5 + 3.2)$$

$$= 4.35 \text{ psi}$$

σ_{max} = maximum bending stress

$$= \frac{\beta\ P_{av}\ b^2}{t^2}$$

$$= \frac{0.500 \times 4.35 \times 16.5^2}{0.1875^2}$$

$\sigma_{max} < S$

i.e. $16900 < 18840$

Condition Satisfied

Maximum deflection of panel

$\alpha = 0.029$ (from Graph #9)

d_{max} = maximum deflection

$$= \frac{\alpha \, P_{av} \, b^4}{E \, t^3}$$

$$= \frac{0.029 \times 4.35 \times 16.5^4}{29.0 \times 10^6 \times 0.1875^3}$$

$= 0.049$ in.

$d_{max} < t_a/2$

i.e. $0.049 < 0.094$

Condition Satisfied

b) <u>Upper Panel</u>

For: $$l = t\sqrt{\frac{2S}{P}}$$

$$= t\sqrt{\frac{2X \; 18840}{3.2}}$$

t	1/8	3/16	1/4	5/16	3/8
l	13.56	20.35	37.13	33.91	40.69

Try using 3/16 TK plate with 6 panels spaced at 22"

$\frac{h}{b} = \frac{60}{22}$

$= 2.73$

$\beta = 0.275$ (from Graph #4)

$\alpha = 0.0177$ (from Graph #4)

σ_{max} = maximum bending stress

$$= \frac{\beta P b^2}{t^2}$$

$$= \frac{0.275 \times 3.2 \times 22^2}{0.1875^2}$$

$$= 12100 \text{ psi}$$

$\sigma_{max} < S$

i.e. 12100 < 18840

Condition Satisfied

d_{max} = maximum deflection

$$= \frac{\alpha P b^4}{E t^3}$$

$$= \frac{0.0177 \times 3.2 \times 22^4}{29.0 \times 10^6 \times 0.1875^3}$$

$$= 0.07 \text{ in.}$$

$d_{max} < 0.094$

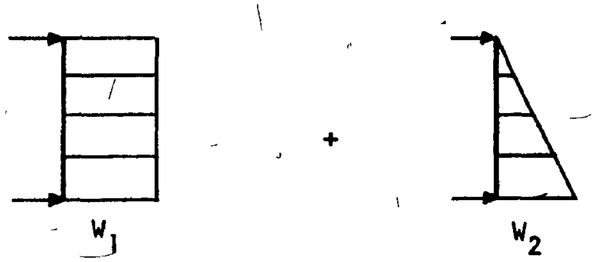
Condition Satisfied

9.3.4 Design of Vertical Stiffener

A) Lower Panel

Try using 3/8" x 4" bar.

By principle of superposition, each stiffener can be considered subject to a uniform load and a traingular load.



$W_1 = 16.5 \times 3.2$

$= 53 \quad lb/in$

$W_2 = 16.5 \times 2.3$

$= 38 \quad lb/in$

M_{max} = maximum bending moment

$$= \frac{W_1 h^2}{8} + 0.0641 W_2 h^2$$

$$= (\frac{53}{8} + 0.0641 \times 38) \times 42^2$$

$= 16000$ lb-in

Z = section modulus

$$= \frac{0.375 \times 4^2}{6}$$

$= 1.00$ in

σ_{max} = maximum bending stress

$$= \frac{M_{max}}{Z}$$

$$= \frac{16000}{1.0}$$

$= 16000$ psi

$\sigma_{max} < S$

i.e. $16000 < 18000$

Condition Satisfied

B) Upper Panel

Try using 3/8" x 4" bar

$W = 22 \times 3.2$

$= 70$ lb/in

M_{max} = maximum bending moment

$= 0.064\ Wh^2$

$= 0.0641 \times 70 \times 60^2$

$= 16200$ lb-in

σ_{max} = maximum bending stress

$= \frac{M}{Z}$

$= \frac{16200}{1.0}$

$= 16200$ psi

$\sigma_{max} < S$

i.e. $16200 < 18000$

Condition Satisfied

9.3.5 Design of Intermediate Horizontal Stiffener

P_h = intermediate horizontal stiffener reaction

$= 0.3316\ Ph$

$= 0.3316 \times 5.5 \times 102$

$= 186$ lb/in.

F = axial force on length side of stiffener

$= 186 \times 60 \times 1/2$

$= 5580$ lb

Try using wide flange beam 8 x 17 ASTM-A36

M_h = bending moment at intermediate horizontal stiffener

$$= \frac{R_h}{12} \times \frac{b^3 + a^3}{b + a}$$

$$= \frac{186}{12} \times \frac{132^3 + 60^3}{132 + 60}$$

$$= 203000 \text{ lb-in}$$

σ_b = bending stress

$$= \frac{M}{Z}$$

$$= \frac{203000}{14.1}$$

$$= 14400 \text{ psi}$$

σ = axial stress

$$= \frac{F}{A}$$

$$= \frac{5580}{5.01}$$

$$= 1100 \text{ psi}$$

Sy = minimum yield strength

$$= 36000 \text{ psi}$$

According to CSA S16 Standard - Steel Structures for Buildings

S_t = allowable stress in tension

$$= 0.60 \text{ Sy}$$

$$= 0.60 \times 36000$$

$$= 21600 \text{ psi}$$

S_{bt} = allowable bending stress in tension

= 0.66 Sy

= 0.66 x 36000

= 23760 psi

S_{bc} = allowable bending stress in compression

$$= 1.15\ S_{bt}\ (1 - \frac{0.28\ S_{bt}}{S_1}) \qquad \text{if } S_{bt} > S_1 \geq 2/3\ S_{bt}$$

$$\text{or} \quad = S_1 \qquad \text{if } S_1 < 2/3\ S_{bt}$$

where

$$S_1 = \sqrt{S_2^{\ 2} + S_3^{\ 2}}$$

$$S_2 = \frac{12 \times 10^6}{1D/A_f}$$

$$S_3 = \frac{149 \times 10^6}{(1/r_t)^2}$$

A_f = area of compression flange

= 1.617 in^2

1 = unsupported length of compression flange

= 132 in

D = depth of section

= 8.00 in

r_t = radius of gyration about its axis of symmetry of a tee section comprising the compression flange and 1/16 of the web

= 1.40 in.

Therefore, we have

$$S_2 = \frac{12 \times 10^6 \times 1.617}{132 \times 8} = 18400 \text{ psi}$$

$$S_3 = \frac{149 \times 10^6 \times 1.40^2}{132^2}$$

$$= 16760 \text{ psi}$$

$$S_1 = \sqrt{18400^2 + 16760^2}$$

$$= 24900 \text{ psi}$$

$$> 2/3\ S_{bt}$$

$$S_{bc} = 1.15 \times 23760 \ (1 - \frac{0.28 \times 23760}{24900})$$

$$= 20000 \text{ psi}$$

For combined axial and bending stress

$$\frac{\sigma}{S_t} + \frac{\sigma_b}{S_{bc}} = \frac{1100}{21600} + \frac{14400}{20000}$$

$$= 0.77$$

$$< 1.0$$

Condition Satisfied

9.3.6 Design of Bottom Horizontal Stiffener

R_b = bottom reaction

= 0.1312 Ph

= 0.1312 x 5.5 x 102

= 74 lb/in

F = axial force on length side of stiffener

= 74 x 60 x 1/2

= 2220 lb

Try using channel 8 x 11.5 ASTM-A36

$$M_b = \frac{R_b}{12} \times \frac{b^3 + a^3}{b + a}$$

$$= \frac{74}{12} \times \frac{132^3 + 60^3}{132 + 60}$$

$= 80800$ lb-in

σ_b = bending stress

$$= \frac{M}{Z}$$

$$= \frac{80800}{8.14}$$

$= 9900$ psi

σ = axial stress

$$= \frac{F}{A}$$

$$= \frac{2220}{3.38}$$

$= 660$ psi

Sy = minimum yield strength

$= 36000$ psi

S_t = allowable stress in tension

$= 21600$ psi

S_{bt} = allowable bending stress in tension

$= 23760$ psi

$$S_2 = \frac{12 \times 10^6}{1D/A_f}$$

$$= \frac{12 \times 10^6 \times 0.882}{132 \times 8}$$

$= 10000$ psi

$$S_3 = \frac{149 \times 10^6}{(1/r_t)^2}$$

$$= \frac{149 \times 10^6 \times 0.625^2}{132^2}$$

$= 3340$ psi

$$S_1 = \sqrt{S_2^2 + S_3^2}$$

$$= \sqrt{10000^2 + 3340^2}$$

$$= 10550 \text{ psi}$$

$$< 2/3\ S_{bt}$$

$$S_{bc} = 10500 \text{ psi}$$

For combined axial and bending stress

$$\frac{\sigma}{S_t} + \frac{\sigma_b}{S_{bc}} = \frac{660}{21600} + \frac{9900}{10550}$$

$$= 0.97$$

$$< 1.0$$

Condition Satisfied

9.3.7 Top Edge Stiffener

The top edge stiffener in a closed tank is not subject to bending. The sidewall reactions are taken up by the roof plate. A steel angle 4 x 4 x 3/8 is suggested for the top edge stiffener.

9.3.8 Design of Tank Bottom Plate

Try using 3/8" x 3" bar as stiffener along center of the length side for a 1/4" TK. base plate.

$$\frac{h}{b} = \frac{30}{24}$$

$$= 1.25$$

$\beta = 0.395$ (from Graph #9)

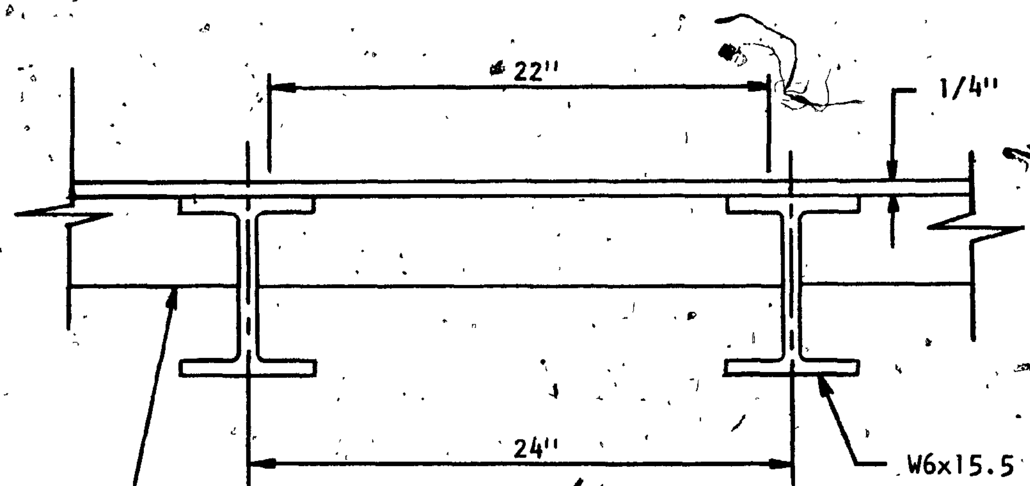


Figure 11. Bottom Beams Spacing

σ_{max} = maximum bending stress

$$= \frac{\beta P b^2}{t^2}$$

$$= \frac{0.395 \times 5.5 \times 24^2}{0.25^2}$$

$$= 20000 \text{ psi}$$

$\sigma_{max} > S$

i.e. $20000 > 18840$

Condition not satisfied

This theoretical bending stress seems to be too high. However, due to the large support of the 6 x 15.5 wide flange beam, an effective stiffener spacing of 22" is acceptable. (See Figure 11.)

$$\frac{h}{b} = \frac{30}{22}$$

$$= 1.36$$

$\beta = 0.42$ (from Graph #9)

$\alpha = 0.021$ (from Graph #9)

σ_{max} = maximum bending stress

$$= \frac{0.42 \times 5.5 \times 22^2}{0.25^2}$$

$$= 17900 \text{ psi}$$

$\sigma_{max} < S$

i.e. $17900 < 18840$

Condition Satisfied

d_{max} = maximum deflection

$$= \frac{\alpha \; P \; b^4}{E \; t^3}$$

$$= \frac{0.021 \times 5.5 \times 22^4}{29.0 \times 10^6 \times 0.25^3}$$

$$= 0.059 \text{ in}$$

$d_{max} < t_a/2$

i.e. $0.059 < 0.125$

Condition Satisfied

<u>Size of Stiffening rib</u>

W = unit load

$$= 30 \times 5.5$$

$$= 165 \text{ lb/in}$$

M_{max} = maximum bending moment

$$= \frac{Wl^2}{12}$$

$$= \frac{165 \times 24^2}{12}$$

$$= 7920 \text{ lb-in}$$

Z = section modulus

$$= \frac{0.375 \times 3.00^2}{6}$$

$$= 0.56 \text{ in}^3$$

σ_{max} = maximum bending stress

$$= \frac{M}{Z}$$

$$= \frac{7920}{0.56}$$

= 14100 psi

$\sigma_{max} < S$

i.e. 14100 < 18000

Condition Satisfied

9.3.9 Design of Tank Roof

P_l = live load

= 25 psf

= 0.174 psi

P_d = dead load

= 0.053 psi for 3/16" plate

P = total load

= 0.174 + 0.053

= 0.227 psi say using P = 0.25 psi

$$l = t\sqrt{\frac{2S}{P}}$$

$$= 0.1875\sqrt{\frac{2x\ 18840}{0.227}}$$

= 76.4 in

$$\frac{h}{b} = \frac{132}{60}$$

= 2.2

$\beta = 0.500$ (from graph #9)

$\alpha = 0.0282$ (from Graph #9)

σ_{max} = maximum bending stress

$$= \frac{\beta\ P\ b^2}{t^2}$$

$$= \frac{0.500 \times 0.250 \times 60^2}{0.1875^2}$$

$= 12800$ psi

$\sigma_{max} < S$

i.e. $12800 < 18840$ psi

Condition Satisfied

d_{max} = maximum deflection

$$= \frac{\alpha\ P\ b^4}{E\ t^3}$$

$$= \frac{0.0282 \times 0.250 \times 60^4}{29.0 \times 10^6 \times 0.1875^3}$$

$= 0.480$ in

$d_{max} > t_a/2$

i.e. $0.480 > 0.094$

Condition not satisfied

However, the roof plate may be considered to carry part of the load in direct tension (diaphragm tension). The top frame which must take up this tension is held in place by the fluid hydrostatic pressure. Therefore, thin plate with large deflection analysis may be applicable in this case.

$$\frac{P\ b^4}{E\ t^4} = \frac{0125 \times 60^4}{29.0 \times 10^6 \times 0.1875^4}$$

$$= 90.4$$

$$\frac{a}{b} = \frac{132}{60}$$

$$= 2.2$$

From Reference 5, article 10.11 for the case of edges held but not fixed:

$$\frac{d}{t} = 1.54$$

$$\frac{\sigma_d\ b^2}{E\ t^2} = 6.39$$

$$\frac{\sigma\ b^2}{E\ t^2} = 13.80$$

$$\sigma_d = \frac{6.39\ Et^2}{b^2}$$

$$= \frac{6.39 \times 29.0 \times 10^6 \times 0.1875^2}{60^2}$$

$$= 1810 \text{ psi}$$

σ = total stress

$$= \frac{13.80\ Et^2}{b^2}$$

$$= \frac{13.80 \times 29.0 \times 10^6 \times 0.1875^2}{60^2}$$

$$= 3910 \text{ psi}$$

$$\sigma < S$$

i.e. 3910 < 18840 psi

Condition Satisfied

d = deflection of panel

= 1.54 t

= 1.54 x 0.1875

= 0.289 in - acceptable

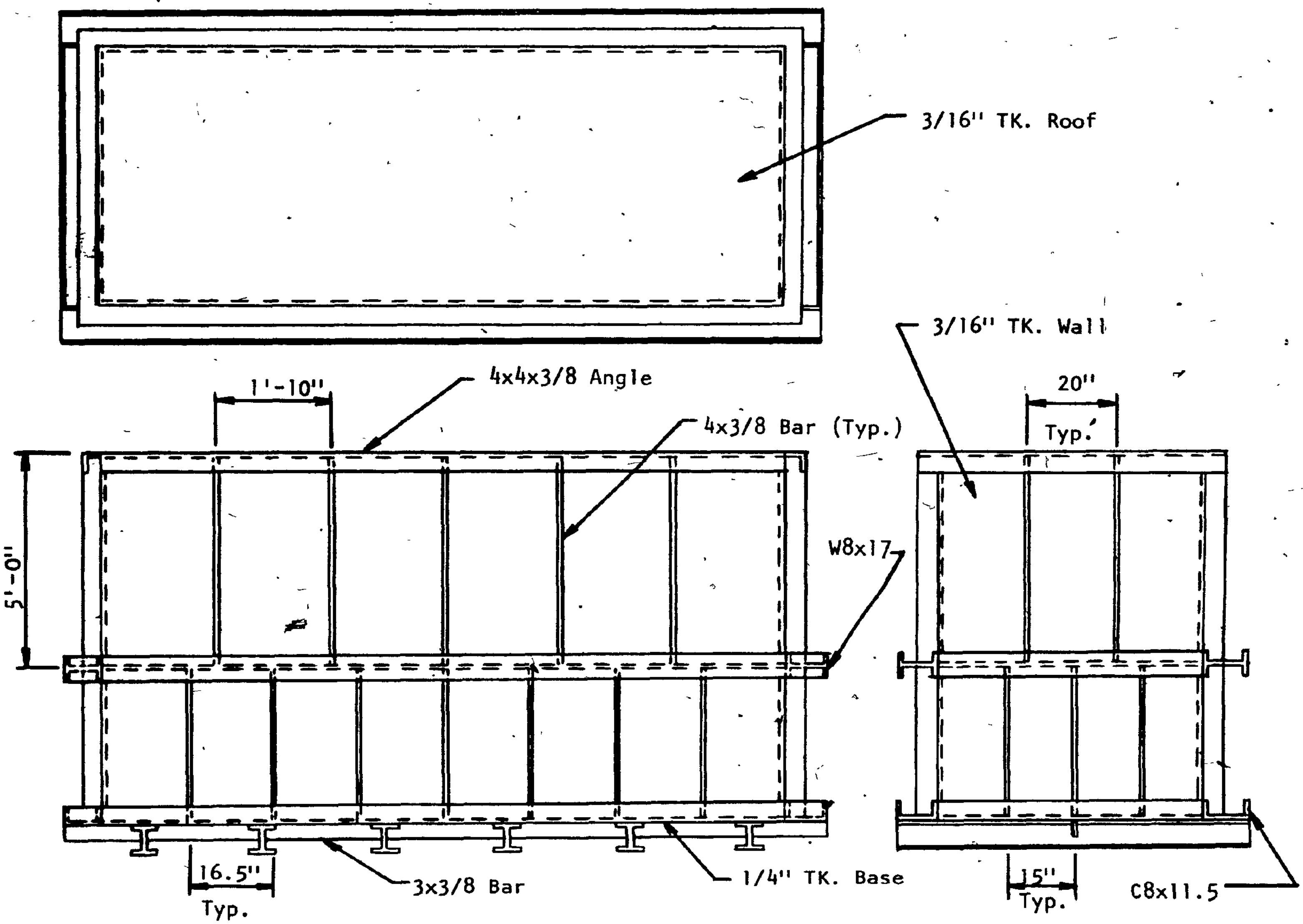


Figure 12. General Arrangement of Storage Tank

10.0 CONCLUSIONS

This report has outlined a method for the design of rectangular storage tanks based on flat-plate theory and flexural analysis straight beam.

Actual detailed analysis by numerical computation or by experimental analysis is very complex and tedious. Several rectangular storage tanks of various sizes have been successfully built for the chemical industry using the design approach described in this thesis.

11.0 REFERENCES

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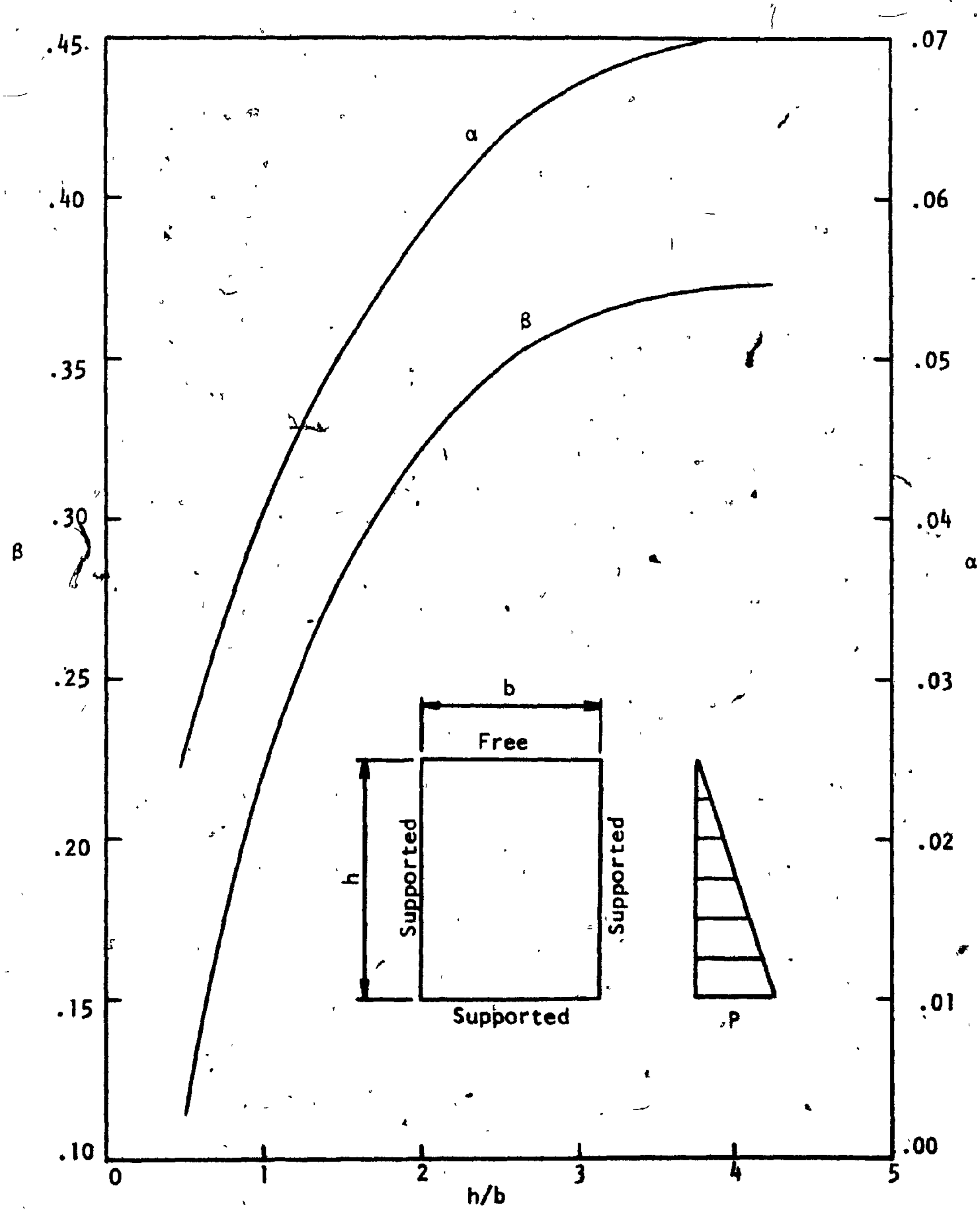
12.0 APPENDIX I

Numerical Graphs

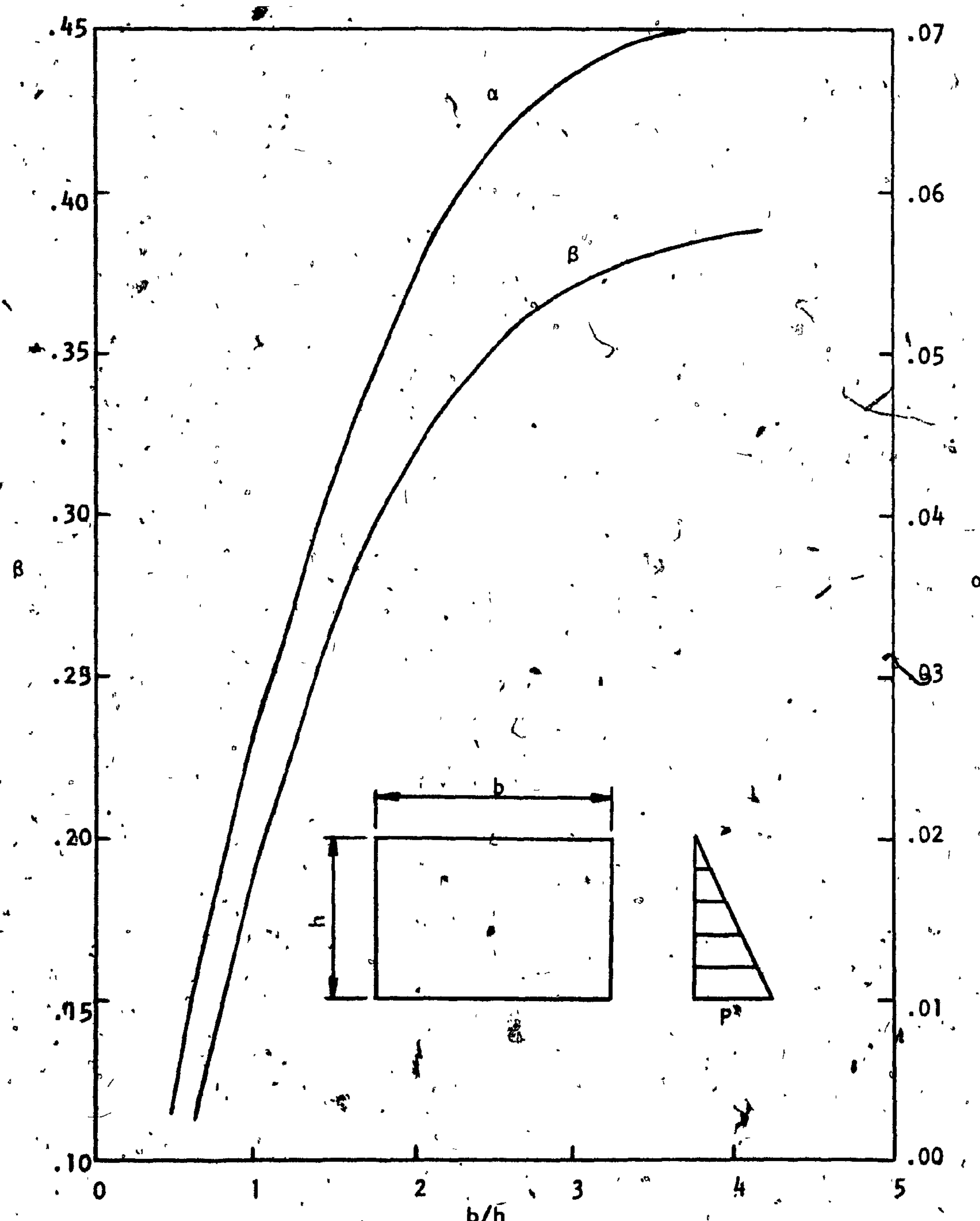
Graph 1

Top edge free, simply supported other edges.
Load increasing uniformly from zero at top edge
to a maximum P at bottom edge.
(From Reference 5, Table 26 Case 2d)



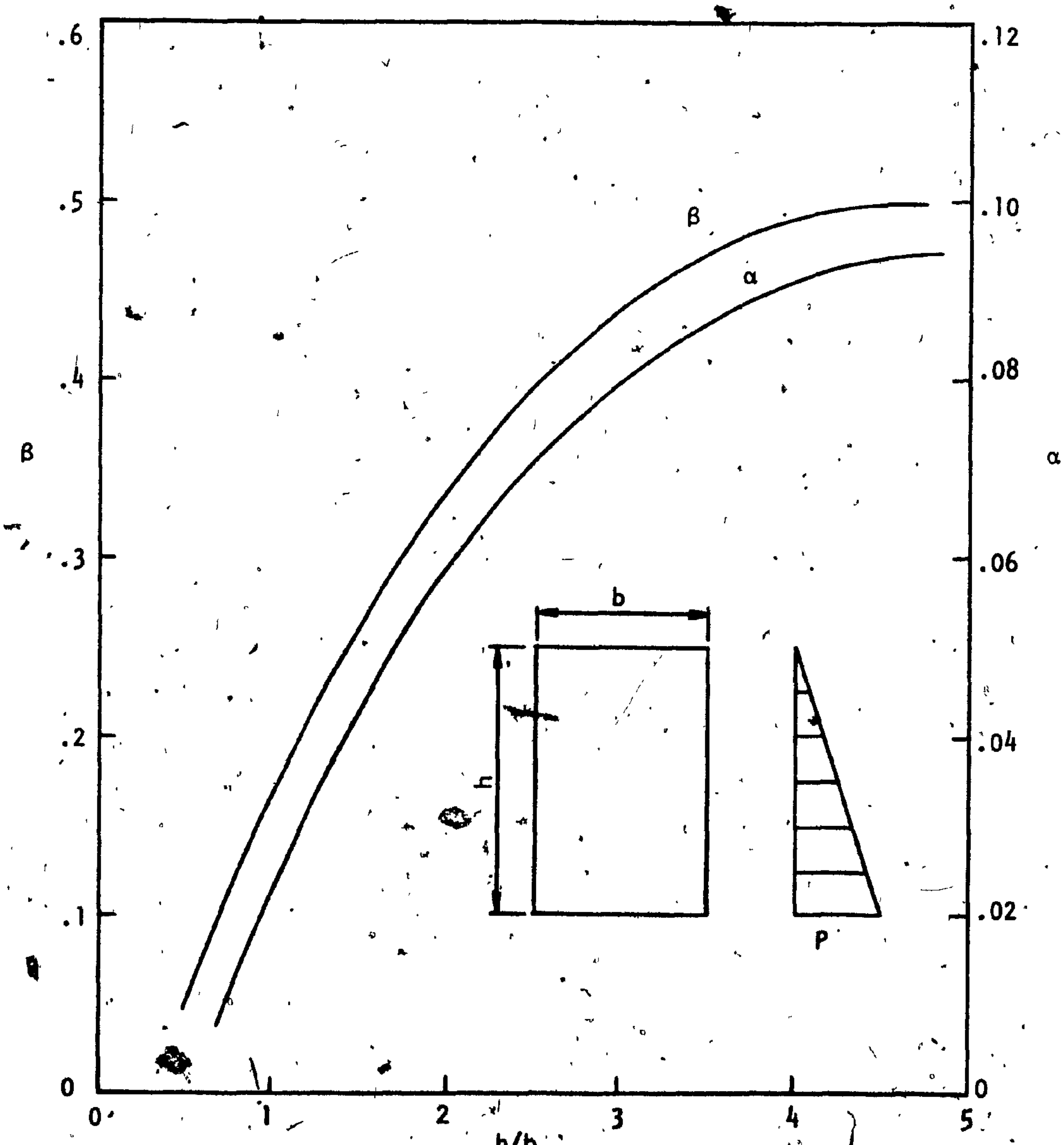
Graph 2

All edges simply supported. $h < b$
Load increasing uniformly from zero at top edge to maximum P at bottom edge.
(From Reference 5, Table 26 Case 1e)



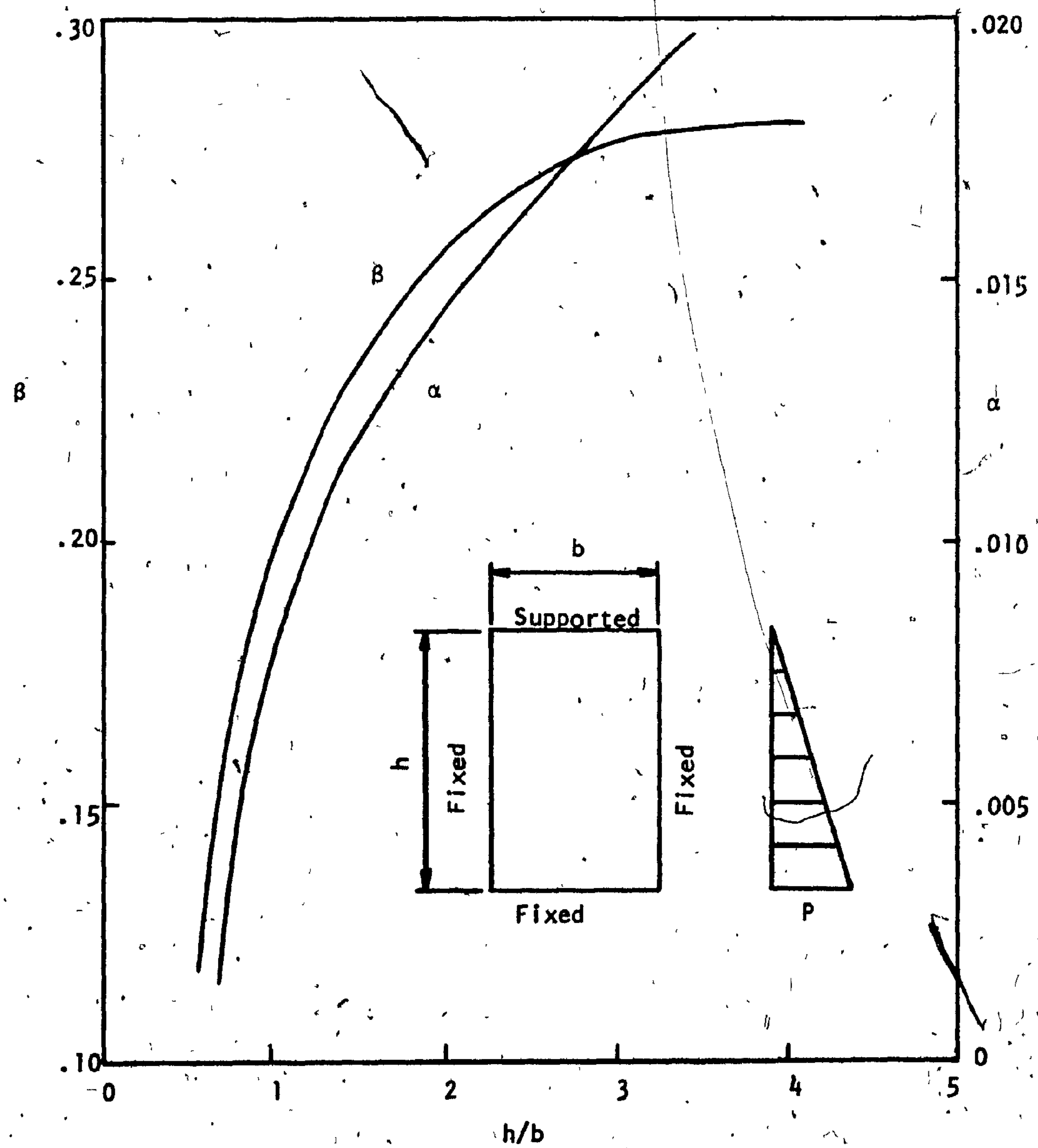
Graph 3

All edges simply supported. h > b
Load increasing uniformly from zero at top edge to a maximum P at bottom edge.
(From Reference 5, Table 26 case 1d)



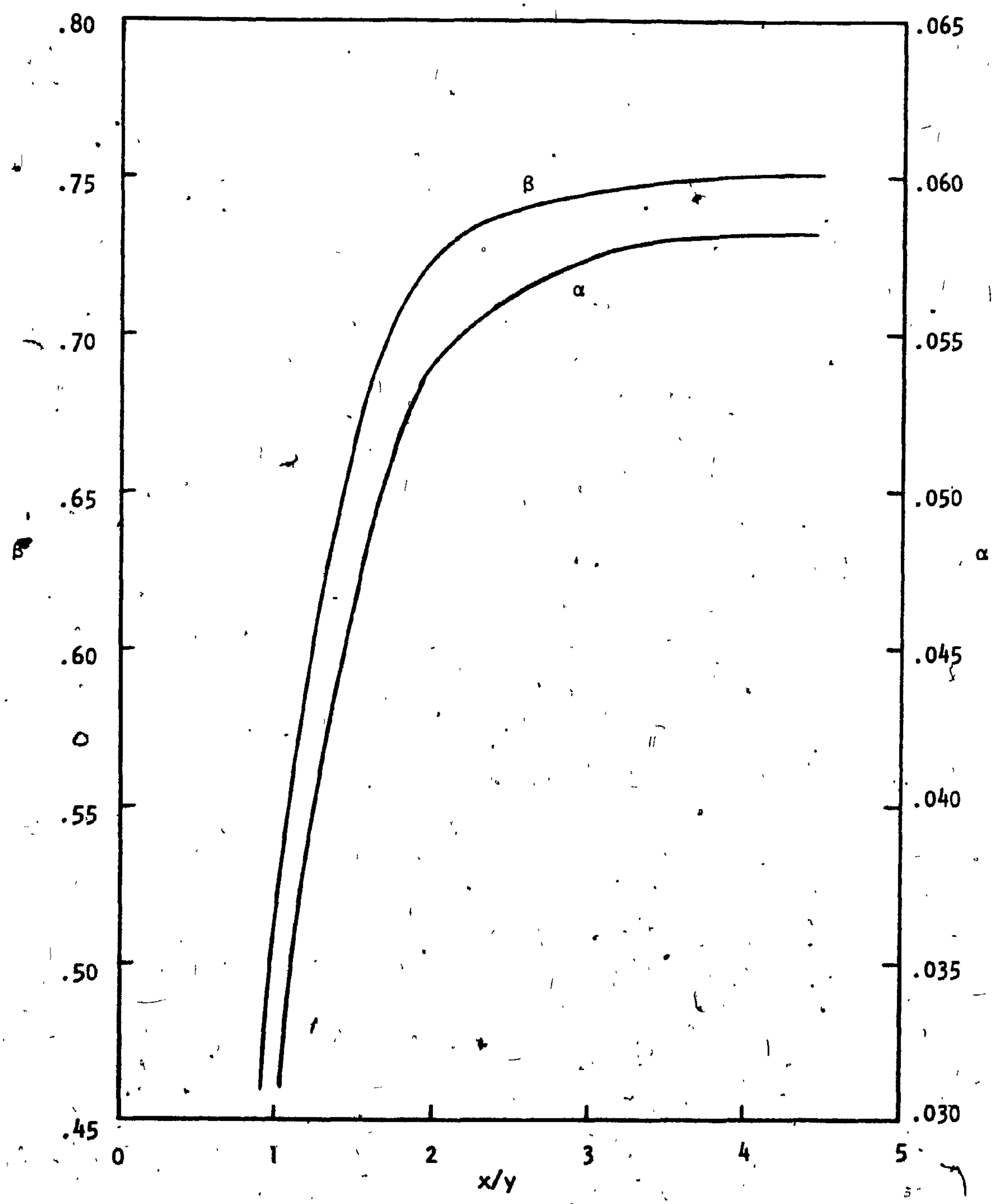
Graph 4

Top edge simply supported, other edges fixed.
Load increasing uniformly from zero at top edge
to a maximum P at bottom edge.
(From Reference 5 Table 26 Case 9d and 8d)



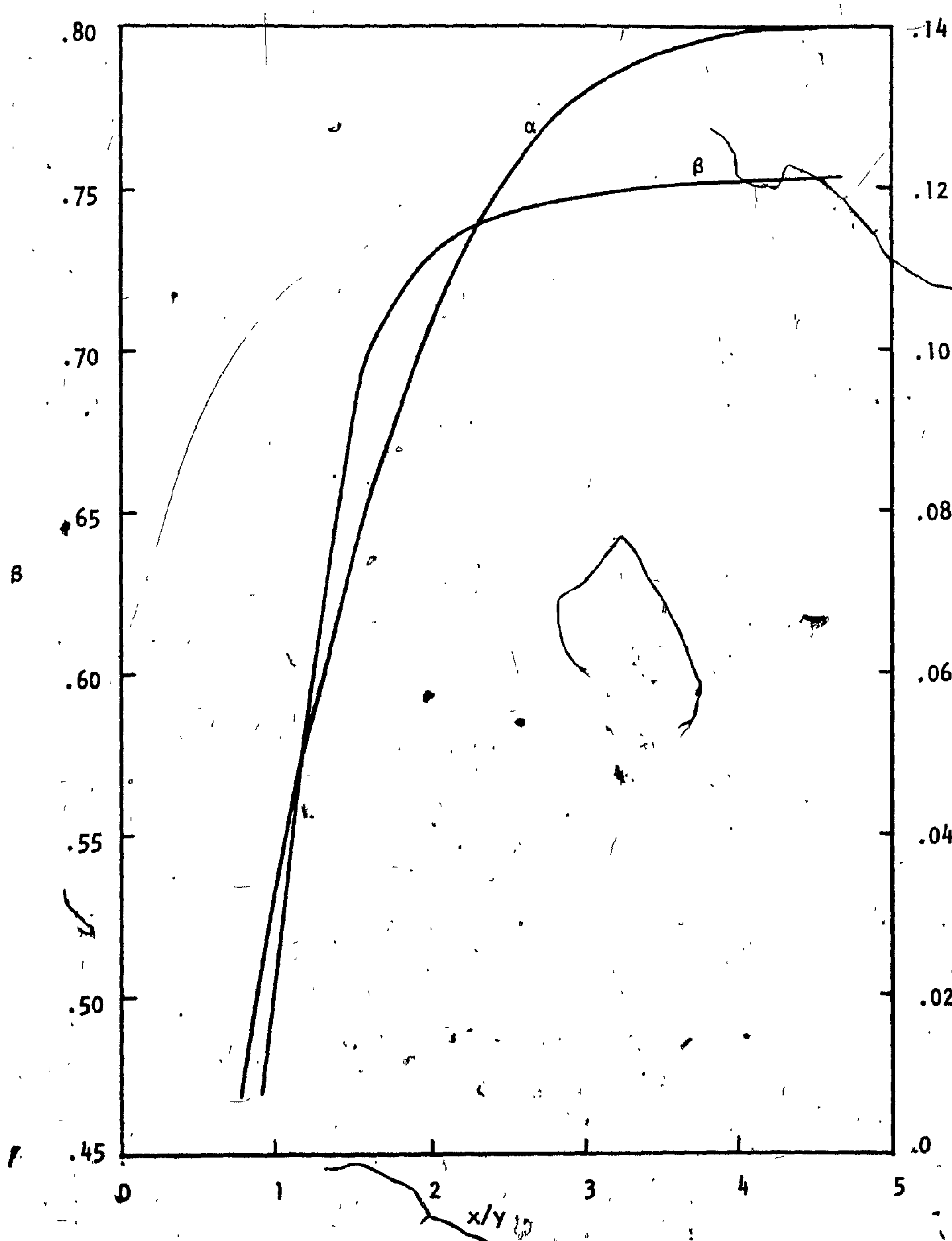
Graph 5

Three edges simply supported and one long edge (x) fixed.
Uniformly distributed load.
(From Reference 5, Table 26 Case 4a)



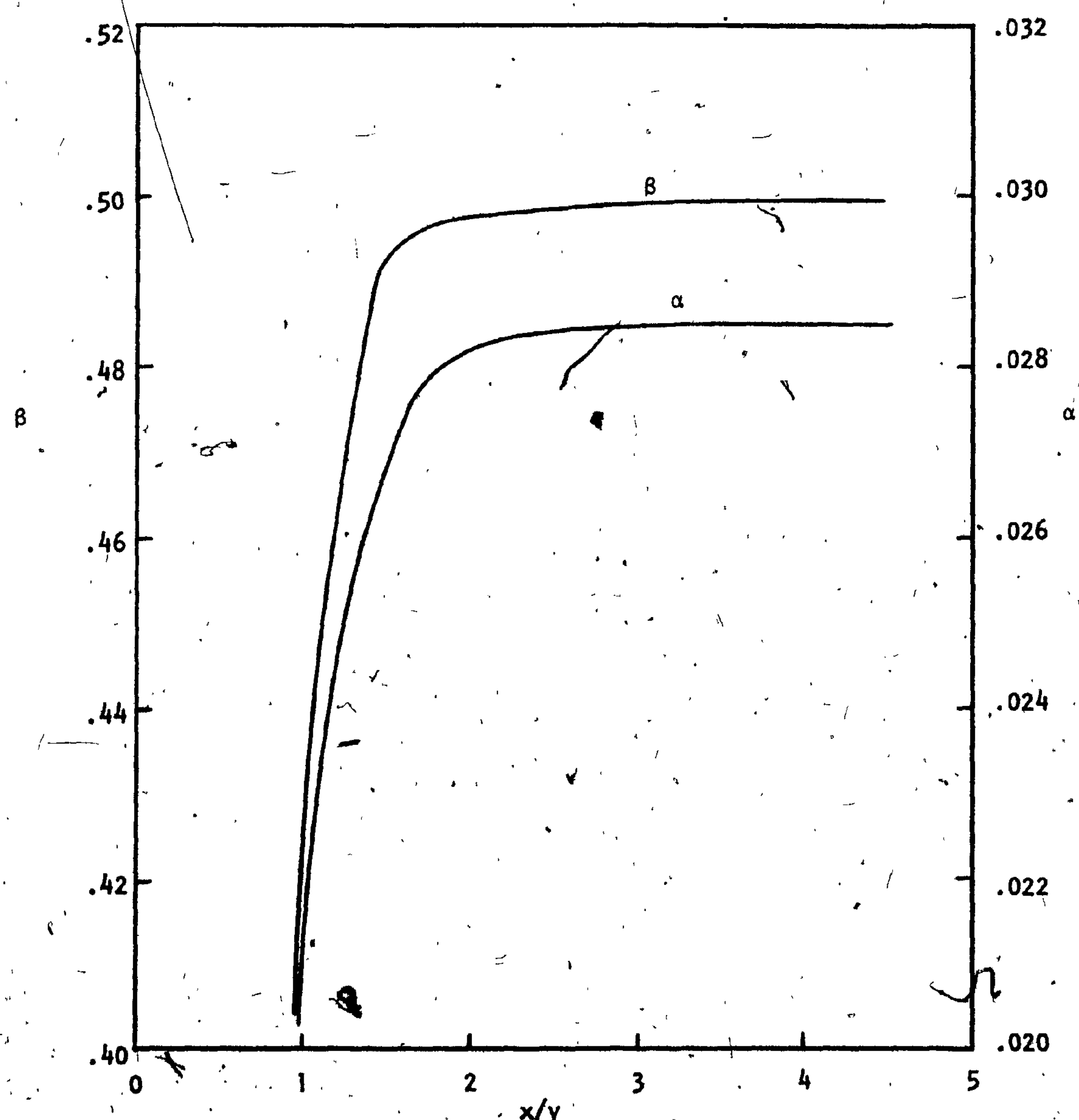
Graph 6

Three edges simply supported and one short edge (y) fixed.
Uniformly distributed load.
(From Reference 5, Table 26 Case 3a)



Graph 7

Two long edges (x) fixed and two short edges (y) simply supported.
Uniformly distributed load.
(From Reference 5, Table 26 Case 6a)



Graph 8

Two long edges (x) simply supported and two short edges (y) fixed.
Uniformly distributed load.
(From Reference 5, Table 26 Case 5a)